Sample Questions

1. A school has 800 students. Of these, 400 students drink milk and 300 students drink coffee. 150 students drink both milk and coffee. How many students drink neither milk nor coffee?

a. 175
b. 250
c. 325
d. 450

2. If $-10 \leq 3x + 4 < 20$, what would be the interval notation of the value of x?

a. $(-14/3, \infty)$
b. $(\infty, 16/3)$
c. $[-14/3, 16/3]$
d. $[-14/3, 16/3)$

3. If $f(x) = x^2 - 2x + 12$ and $g(x) = (x - 1)$, what is the value of function $f(g(x))$?

a. $x^3 - 4x$
b. $x^2 - 4x + 15$
c. $x^2 - 8x + 17$
d. $x^2 + 4x - 15$

4. If $f(x) = 3x + 2$, what is $f^{-1}(x)$?

a. $2x + 3$
b. $(x-2) / 3$
c. $(3x - 2) / 3$
d. $- 2/3$

5. Simplify the following:

$$\frac{(4x^{-3})(10x^{3})}{(5y^{-56})(9y^{56})}$$

a. $\frac{5}{6}$
b. $\frac{8}{9}$
c. $\frac{10}{15}$
d. $\frac{7}{3}$

6. What is the slope of the line passing through (2, 5) and (-1, -4) points?

a. 5
b. 6/5
c. 3
d. -1/2

7. Find $\lim_{x \to 2} \frac{x^2 - x - 4}{2x^2 + 4x + 5}$

a. -2/21
b. 10
c. -5/3
d. 35

8. 255 feet of fencing is needed to enclose a rectangular-shaped back yard. If the yard is 32 feet wide, what is the length?

a. 121 ft
b. 36.56 ft
c. 95.5 ft
d. 0.521 ft

9. George can run 25 miles in the same amount of time that his brother, Alan, can run 15 miles. If George runs 3 miles per hour faster than Alan, how fast does Alan run?

a. 10
b. 4.5
c. 15
d. 7.6

10. One pipe can fill a tank in 30 minutes. The tank can be drained by an other pipe in 70 minutes. If both pipes are opened, how long will it take to fill the tank?

a. 21
b. 52.5
c. 36
d. 40.6

Answers

1. (b) These types of problems can be solved by using the SET THEORY. There are few terminologies and equations you should keep in mind to solve these problems.

1. (U) = It is known as universal set. The universal set depends on the context.

2. $A \subset B$ = When every element of set A also belongs to set B, then set A is said to be the subset of B. For example,

$A = \{1, 2, 3\}$ and $B = \{0, 1, 2, 3, 4\}$

3. When $A \subset B$ and $B \subset A$, then A = B or they are called equal sets.

4. $A \cup B$ = The set consisting of all elements which are in A or in B is called the union of A and B, and is denoted by $A \cup B$. For example,

$A = \{1, 2, 3\}$ and $B = \{0, 1, 2, 3, 4\}$

$A \cup B = \{0, 1, 2, 3, 4\}$

5. $A \cap B$ = The set consisting of all elements which are common in both A and B is called the intersection of A and B, and is denoted by $A \cap B$. For example,

$A = \{1, 2, 3\}$ and $B = \{0, 1, 2, 3, 4\}$

$A \cap B = \{1, 2, 3\}$

6. (A') = The set consisting of all those elements of U which are not in A is called the component of A and is denoted by A'. For example,

$U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2\}$ then,

$A' = \{3, 4, 5\}$

7. De Morgan's Laws:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

Let A be the set of students drinking milk, then n(A) = 400

Let B be the set of students drinking coffee, then n(B) = 300

The set of students drinking both milk and coffee is $A \cap B$, therefore $n(A \cap B) = 150$

Finally, let the set of students of the school be U, therefore n(U) = 800

The set of students drinking neither coffee nor milk should be $A' \cap B' = (A \cup B)'$

$$\begin{aligned} n(A \cup B)' &= n(U) - n(A \cup B) \\ &= n(U) - [n(A) + n(B) - n(A \cap B) \\ &= 800 - 400 - 300 + 150 \\ &= 250 \end{aligned}$$

2. (d) Subtract 4 from all parts of the inequality, and then divide everything by 3 to find out the value of x.

$$\begin{aligned} -10 \le 3x + 4 &< 20 \\ -10 - 4 \le 3x + 4 - 4 &< 20 - 4 \\ -14 \le 3x &< 16 \\ -\frac{14}{3} \le x &< \frac{16}{3} \end{aligned}$$

Therefore, the answer in interval form should be [-14/3, 16/3).

3. (b) To solve this problem, we have to substitute x for g(x) in f(x).

$f(g(x)) = f(x - 1) = (x-1)^2 - 2(x - 1) + 12$

$= x^2 - 2x + 1 - 2x + 2 + 12$
$= x^2 - 4x + 15$

4. (b) $f(x) = y$,

$y = 3x + 2$
$x = 3y + 2$ (replacing x an y variables)
$x - 2 = 3y$
$y = f^{-1}(x) = \frac{x - 2}{3}$

5. (b) $\frac{(4x^{-3})(10x^{3})}{(5y^{-56})(9y^{56})}$ therefore:

$= \frac{4 \times 10}{9 \times 5} = \frac{8}{9}$

6. (c) The slope of the line can be calculated by using the following equation:

$m = \frac{y_2 - y_1}{x_2 - x_1}$, where m is the slope.

$m = \frac{-4 - 5}{-1 - 2}$

$m = \frac{-9}{-3}$

$m = 3$

7. (a) We have $\lim_{x \to 2} \frac{x^2 - x - 4}{2x^2 + 4x + 5}$

$\lim_{x \to 2} \frac{x^2 - x - 4}{2x^2 + 4x + 5}$

$\lim_{x \to 2} \frac{2^2 - 2 - 4}{2(2)^2 + 4(2) + 5}$

$\lim_{x \to 2} \frac{-2}{21}$

8. (c) The perimeter of a rectangle can be calculated by using the following equation:

$P = 2L + 2W$, where P = Perimeter
L = Length
W = Width

$P = 2L + 2W$
$255 = 2L + 2(32)$
$255 - 64 = 2L$
$191 = 2L$, therefore:
$L = 95.5$ ft

9. (b) Let r = Alan's rate
and r + 3 = George's rate

$t = \frac{d}{r}$ where
t = time
d = distance
r = rate

Since their times are equal, we can say:

$\frac{15}{r} = \frac{25}{r + 3}$

$15 (r + 3) = 25r$
$15r + 45 = 25r$
$10r = 45$

$r = 4.5$ miles/hr = Alan's rate
$r + 3 = 4.5 + 3 = 7.5$ miles/hr = George's rate

10. (b) Let x be the number of minutes required to fill a tank when both pipes are opened, therefore:

	Minutes to fill tank	Tank filled in 1 minute
Pipe-1	30	1/30
Pipe-2	70	1/70
Together	x	1/x

Since pipe 1 and pipe 2 worked against each other, therefore:

$$\frac{1}{30}-\frac{1}{70}=\frac{1}{x}$$

$$70x-30x=2100$$

$$40x=2100$$

$$x=52.5\,\text{min}\,utes$$

Thus if both pipes are opened, it will take 52.5 minutes to fill the whole tank.